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2007 J. Phys.: Condens. Matter 19 236212

(http://iopscience.iop.org/0953-8984/19/23/236212)

IOP PUBLISHING JOURNAL OF PHYSICS: CONDENSED MATTER

J. Phys.: Condens. Matter **19** (2007) 236212 (8pp) doi:10.1088/0953-8984/19/23/236212

Andreev reflection in a mesoscopic hybrid four-terminal Rashba ring

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Received 16 November 2006, in final form 16 March 2007
Published 11 May 2007
Online at stacks.iop.org/JPhysCM/19/236212

Abstract

The transport properties of a mesoscopic hybrid four-terminal Rashba ring are investigated along the lines of Blonder, Tinkham and Klapwijk and our attention is focused on the influence of Andreev reflection. Although the resonant peak and zero conductance resulting from spin-dependent quantum interference are destroyed by incoherent addition in this four-terminal structure, the total longitudinal electron conductance is still enhanced by the Andreev contribution. However, the contribution of the Andreev reflection to the spin Hall conductance in this mesoscopic Rashba ring is negative and the total spin Hall conductance is reduced. These results are robust to an external magnetic flux.

1. Introduction

The development of semiconductor spintronics provides the hope of using spin, in addition to charge, for quantum information processing [1–3]. The basis of this application is generation of spin-polarized current, and, in several proposals presented to surmount this problem, taking advantage of the spin–orbit (SO) coupling in semiconductors [4–7] has attracted a lot of attention. Due to inversion asymmetry of the confining potential for a two-dimensional electronic gas (2DEG) in semiconductor heterostructure, the Rashba SO coupling [8] plays an important role in electronic transport. The Hamiltonian of a 2DEG with the Rashba SO coupling is [8]

$$H = \frac{\vec{p}^2}{2m^*} + \frac{\alpha}{\hbar}(\vec{\tau} \times \vec{p}) \cdot \vec{e}_z + V_{\mathrm{conf}}(x, y). \tag{1}$$

Here, $\vec{\tau}$ is the Pauli operator, α the strength of the Rashba SO coupling and V_{conf} the potential confining electrons to a mesoscopic region within the 2DEG. In this system, an electron flowing in the longitudinal direction is subjected to a transverse spin 'force' [9, 10], which can be extracted within the Heisenberg picture as $\vec{f} = \frac{2\alpha^2 m^*}{\hbar^3}(\vec{p} \times \vec{e}_z)\hat{\tau}^z - \vec{\nabla} V_{\mathrm{conf}}(x, y)$. The direction of this 'force' depends on the electronic spin polarized on the z-axis, and oscillates on the scale of the spin precession length $L_{\mathrm{SO}} = \frac{\pi\hbar^2}{2m^*\alpha}$ since $|\uparrow\rangle$ and $|\downarrow\rangle$ are not the eigenstates of the system. When an unpolarized electronic current flows through a mesoscopic

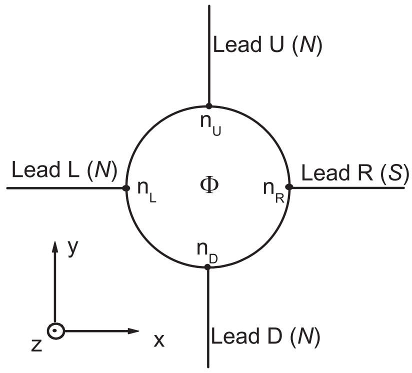

Figure 1. Schematic illustration of the structure.

four-terminal Rashba system, a pure spin current appears in the transverse direction, the so-called spin Hall effect (SHE) [11–17], although it may be erased by disorder in bulk systems. This effect has been demonstrated theoretically in Rashba rings [16, 17] and rectangular Rashba planes with their sizes smaller than the coherence length [13–15].

In these theoretical works, all of the four terminals are normal (N) leads. How about the consequence of introduction of a superconducting (S) lead? In an N–S hybrid system, if the electron energy is restricted in the superconducting gap, the direct tunnelling of electrons through the N–S interface into the S side is forbidden. However, the Andreev reflection can assist the electronic tunnelling [18], and as a spin-up electron tunnels through the N–S interface a Cooper pair can be injected into the S side. As a result, a spin-up hole is reflected back into the N side. Obviously, in a hybrid system with the spin degeneracy lifted by the Rashba SO coupling, the tunnelling of the Andreev-reflected hole back through the structure affects the total spin Hall conductance. From a naive viewpoint, the Andreev reflection should increase the spin Hall conductance since the transverse spin 'acceleration' of the Andreev-reflected spin-up hole is the same as that of the incident spin-up electron due to the momentum conservation in the Andreev reflection. But, as we have said, spin is not a good quantum number in a Rashba system but precesses in its tunnelling process. The contribution of Andreev reflection has to be determined from a more sophisticated analysis. The purpose of the present paper is to numerically study the transport properties of a mesoscopic hybrid four-terminal Rashba ring and clarify the influence of Andreev reflection on the spin Hall conductance in this structure.

For this purpose, we consider a structure schematically illustrated in figure 1, and study the spin Hall conductance in the ballistic transport process by assuming the structural dimension is much smaller than the coherence length. Here, this spin-nondegenerate hybrid system is investigated along the lines of Blonder, Tinkham and Klapwijk (BTK) [19]. Although the resonant peak and zero conductance resulting from spin-dependent quantum interference are destroyed by incoherent addition in this four-terminal structure, the longitudinal electron conductance is still enhanced by the Andreev contribution. However, the contribution of the Andreev-reflected hole to the spin Hall conductance partly offsets that of the incident electron, which explains the negative contribution of the Andreev reflection to the spin Hall conductance in the Rashba ring. These characteristics are robust to an external magnetic flux since the spin 'acceleration' is irrelevant with the flux, although the time-reversal symmetry (TRS) is broken.

The organization of this paper is as follows. In section 2, the theoretical model and calculation method are presented. In section 3, the numerical results are illustrated and discussed. A brief summary is given in section 4.

2. Model and formulas

In the present paper, we consider the ballistic transport through a mesoscopic hybrid four-terminal Rashba ring and investigate the influence of Andreev reflection on the spin Hall conductance. The structure is schematically illustrated in figure 1, where a mesoscopic ring of length M with the SO hopping parameter t_{SO} is connected with four leads where no SO interaction exists. All of these leads are normal except the right-hand one, which is a superconducting lead with the energy gap Δ. In the present paper, we only consider a structure where these leads are symmetrically connected to the ring with tunnelling matrix elements $t_{\mathrm{T}}^{(l)}$ with $l = L, R, U, D$. As a result, $n_L = M/2+1$, $n_R = 1$, $n_U = M/4+1$ and $n_D = 3M/4+1$, respectively, in figure 1. The time-reversal symmetry of the system can be broken by a magnetic flux Φ, which penetrates the ring area. With the z direction perpendicular to the ring plane, the Hamiltonian can be written in the tight-binding representation as [16, 17, 20, 21]

$$H = H_{\mathrm{ring}} + H_{\mathrm{lead}} + H_{\mathrm{T}}, \tag{2}$$

where H_{ring}, H_{lead} and H_{T} are the Hamiltonians of the ring, the leads and the tunnelling between them. They are

$$H_{\mathrm{ring}} = \sum_{n=1\sigma}^{M} \left\{ -t d_{n\sigma}^{\dagger} d_{n+1\sigma} + \imath t_{\mathrm{SO}} \sum_{\sigma'} \left(\cos\varphi_n \hat{\tau}_{\sigma\sigma'}^{x} + \sin\varphi_n \hat{\tau}_{\sigma\sigma'}^{y} \right) d_{n\sigma}^{\dagger} d_{n+1\sigma'} + \mathrm{H.c.} \right\}, \tag{3}$$

$$H_{\mathrm{lead}} = -t \sum_{l} \sum_{n=1\sigma}^{\infty} \left(c_{n\sigma}^{(l)\dagger} c_{n+1\sigma}^{(l)} + \mathrm{H.c.} \right) + \sum_{n=1}^{\infty} \left(\Delta^{*} c_{n\downarrow}^{(R)} c_{n\uparrow}^{(R)} + \Delta c_{n\uparrow}^{(R)\dagger} c_{n\downarrow}^{(R)\dagger} \right) \tag{4}$$

and

$$H_{\mathrm{T}} = -\sum_{l\sigma} \left(t_{\mathrm{T}}^{(l)} c_{1\sigma}^{(l)\dagger} d_{n_l\sigma} + \mathrm{H.c.} \right), \tag{5}$$

where $d_{n\sigma}^{\dagger}$ ($d_{n\sigma}$) and $c_{n\sigma}^{(l)\dagger}$ ($c_{n\sigma}^{(l)}$) are the electronic creation (annihilation) operators of the ring and lead l, respectively, with the spin index $\sigma = \uparrow, \downarrow$ or ± 1. Here, we choose a gauge where the flux Φ enters only as a boundary condition [22], and $d_{N+1\sigma} = \mathrm{e}^{\mathrm{i}\Phi} d_{1\sigma}$. If the ring centre is set as the origin of the coordinate system, $\varphi_n = 2\pi(n - \frac{1}{2})/M$. With the z axis set as the quantization direction, $\hat{\tau}^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\hat{\tau}^y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix}$ and $\hat{\tau}^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. This tight-binding Hamiltonian is obtained from the effective mass Hamiltonian (1) by employing the local orbital basis and has been successfully applied to quasi-1D and 2D structures with the spin Hall effect [13, 14, 16, 17, 21]. It is related to the effective mass Hamiltonian (1) via the relations $t = \hbar^2/(2m^* a^2)$ and $t_{\mathrm{SO}} = \alpha/(2a)$ with a the lattice spacing.

Because of the Rashba SO coupling, the system is spin nondegenerate. The bases of the wavefunction in the leads and the ring are set as $(c_{n\uparrow}^{(l)\dagger}, c_{n\downarrow}^{(l)\dagger}, c_{n\downarrow}^{(l)}, c_{n\uparrow}^{(l)})|0\rangle$ and $(d_{n\uparrow}^{\dagger}, d_{n\downarrow}^{\dagger}, d_{n\downarrow}, d_{n\uparrow})|0\rangle$ with $|0\rangle$ the ground state. When an electron with energy ϵ and spin σ is incident from the left-hand lead to the ring, the corresponding wavefunction in this lead is

$$\Psi_{\sigma}^{(LL)} = \sum_{n=1}^{\infty} \left\{ \mathrm{e}^{-\mathrm{i}q_{\mathrm{e}}n} c_{n\sigma}^{(L)\dagger} + t_{\mathrm{e};\uparrow,\sigma}^{(LL)} \mathrm{e}^{\mathrm{i}q_{\mathrm{e}}n} c_{n\uparrow}^{(L)\dagger} + t_{\mathrm{e};\downarrow,\sigma}^{(LL)} \mathrm{e}^{\mathrm{i}q_{\mathrm{e}}n} c_{n\downarrow}^{(L)\dagger} + t_{\mathrm{h};\downarrow,\sigma}^{(LL)} \mathrm{e}^{-\mathrm{i}q_{\mathrm{h}}n} c_{n\downarrow}^{(L)} + t_{\mathrm{h};\uparrow,\sigma}^{(LL)} \mathrm{e}^{-\mathrm{i}q_{\mathrm{h}}n} c_{n\uparrow}^{(L)} \right\} |0\rangle. \tag{6}$$

Similarly, the transmitted wavefunctions through the upper and lower leads can be written as

$$\Psi_{\sigma}^{(lL)} = \sum_{n=1}^{\infty} \left\{ t_{\mathrm{e};\uparrow,\sigma}^{(lL)} \mathrm{e}^{-\mathrm{i}q_{\mathrm{e}}n} c_{n\uparrow}^{(l)\dagger} + t_{\mathrm{e};\downarrow,\sigma}^{(lL)} \mathrm{e}^{\mathrm{i}q_{\mathrm{e}}n} c_{n\downarrow}^{(l)\dagger} + t_{\mathrm{h};\downarrow,\sigma}^{(lL)} \mathrm{e}^{-\mathrm{i}q_{\mathrm{h}}n} c_{n\downarrow}^{(l)} + t_{\mathrm{h};\uparrow,\sigma}^{(lL)} \mathrm{e}^{-\mathrm{i}q_{\mathrm{h}}n} c_{n\uparrow}^{(l)} \right\} |0\rangle, \tag{7}$$

with $l = U, D$. Without external voltage, the system is in the equilibrium state, and $-2t\cos q_\nu = \pm\epsilon$ in these three normal leads with $\nu = \mathrm{e}$ and h, corresponding to $+$ and $-$, respectively. In the right-hand S lead, the transmitted wavefunction is

$$\begin{aligned}\Psi_\sigma^{(RL)} = \sum_{n=1}^{\infty} \Big\{ & t_{\mathrm{e};\uparrow,\sigma}^{(RL)} \mathrm{e}^{\mathrm{i}k_{\mathrm{e}}n} \left(u c_{n\uparrow}^{(R)\dagger} - v c_{n\downarrow}^{(R)} \right) \\ & + t_{\mathrm{e};\downarrow,\sigma}^{(RL)} \mathrm{e}^{\mathrm{i}k_{\mathrm{e}}n} \left(u c_{n\downarrow}^{(R)\dagger} + v c_{n\uparrow}^{(R)} \right) + t_{\mathrm{h};\downarrow,\sigma}^{(RL)} \mathrm{e}^{-\mathrm{i}k_{\mathrm{h}}n} \left(u c_{n\downarrow}^{(R)} + v c_{n\uparrow}^{(R)\dagger} \right) \\ & + t_{\mathrm{h};\uparrow,\sigma}^{(RL)} \mathrm{e}^{-\mathrm{i}k_{\mathrm{h}}n} \left(u c_{n\uparrow}^{(R)} - v c_{n\downarrow}^{(R)\dagger} \right) \Big\} |0\rangle, \end{aligned} \tag{8}$$

where $-2t\cos k_\nu = \pm\sqrt{\epsilon^2 - \Delta^2}$, $u^2 = \frac{1}{2}(1 + \sqrt{\epsilon^2 - \Delta^2}\epsilon)$ and $v^2 = \frac{1}{2}(1 - \sqrt{\epsilon^2 - \Delta^2}/\epsilon)$. If the wavefunction in the ring is expressed as

$$\Psi_\sigma^{(L)} = \sum_{n=1}^{N} \left\{ a_{\mathrm{e};n\uparrow,\sigma}^{(L)} d_{n\uparrow}^\dagger + a_{\mathrm{e};n\downarrow,\sigma}^{(L)} d_{n\downarrow}^\dagger + a_{\mathrm{h};n\downarrow,\sigma}^{(L)} d_{n\downarrow} + a_{\mathrm{h};n\uparrow,\sigma}^{(L)} d_{n\uparrow} \right\} |0\rangle, \tag{9}$$

the transmission coefficient $|t_{\nu;\sigma',\sigma}^{(lL)}|^2$ through the spin σ' channel of lead l and the electronic or hole occupation probability $|a_{\nu;n\sigma',\sigma}^{(L)}|^2$ of spin σ' on the ring site n can be obtained by directly solving the Schrödinger equation.

In a similar way we can obtain the transmission coefficient $|t_{\nu;\sigma',\sigma}^{(l'l)}|^2$ with $l, l' = L, U, D$. In the linear response regime, the electronic and spin currents can be written as $\delta\hat{J} = \frac{e^2}{h}\sum_{\nu;\sigma',\sigma}\hat{L}_{\nu;\sigma',\sigma}\delta\hat{\mu}$ and $\delta\hat{J}^z = \frac{e}{4\pi}\sum_{\nu,\sigma}(\hat{L}_{\nu;\uparrow,\sigma} - \hat{L}_{\nu;\downarrow,\sigma})\delta\hat{\mu}$, respectively, with $\delta\hat{\mu}$ the small external potentials. Here, for the convenience of calculation, the external potential of the right lead is set as the reference, and $\delta\hat{\mu} = (\delta\mu^{(L)}, \delta\mu^{(U)}, \delta\mu^{(D)})^{\mathrm{T}}$. $\delta\hat{J}$ and $\delta\hat{J}^z$ have the same form as $\delta\hat{\mu}$. The matrix of transport coefficient in these equations is

$$\hat{L}_{\nu;\sigma',\sigma} = -\frac{1 \pm 1}{2}\delta_{\sigma',\sigma}^K \pm \begin{pmatrix} \left\| t_{\nu;\sigma',\sigma}^{(LL)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(LU)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(LD)} \right\|^2 \\ \left\| t_{\nu;\sigma',\sigma}^{(UL)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(UU)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(UD)} \right\|^2 \\ \left\| t_{\nu;\sigma',\sigma}^{(DL)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(DU)} \right\|^2 & \left\| t_{\nu;\sigma',\sigma}^{(DD)} \right\|^2 \end{pmatrix} \tag{10}$$

at zero temperature with $\nu = \mathrm{e}$ and h corresponding to $+$ and $-$, respectively. Here, δ^K is the Kronig delta function.

If an unpolarized electronic current is incident from the left-hand lead, at a certain set of external voltages with $\delta\mu^{(U)}/\delta\mu^{(L)} = V_2$ and $\delta\mu^{(D)}/\delta\mu^{(L)} = V_3$, the transverse electronic current is offset and only pure spin current flows in the transverse direction. The longitudinal electronic and spin Hall conductances of the structure are defined in this situation as $G_{xx} = -\delta J^{(L)}/\delta\mu^{(L)}$ and $G_{\mathrm{sH}}^z = (\delta J^{z(U)} - \delta J^{z(D)})/\delta\mu^{(L)}$, respectively. Written explicitly, $G_{xx} = G_{\mathrm{e};xx} + G_{\mathrm{h};xx}$ with

$$G_{\nu;xx} = -\sum_{\sigma'\sigma}\sum_{m=1}^{3}(\hat{L}_{\nu;\sigma'\sigma})_{1,m}V_m. \tag{11}$$

Here, to write the formula concisely, we set $V_1 \equiv 1$ and omit the unit e^2/h. Similarly, the spin-Hall conductance can be written as $G_{\mathrm{sH}}^z = G_{\mathrm{e;sH}}^z + G_{\mathrm{h;sH}}^z$ and

$$G_{\nu;\mathrm{sH}}^z = \sum_{\sigma'\sigma}\sigma'\sum_{m=1}^{3}\left\{(\hat{L}_{\nu;\sigma'\sigma})_{2,m} - (\hat{L}_{\nu;\sigma'\sigma})_{3,m}\right\}V_m \tag{12}$$

with the unit $e/(4\pi)$ omitted. If all of the four leads are normal and the flux Φ is zero, $V_2 = V_3 = 1/2$ due to the structural symmetry of the ring system [14–16].

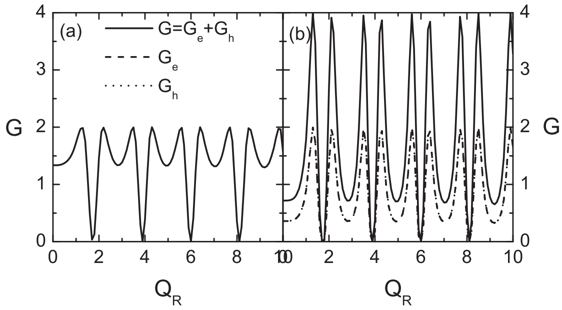

Figure 2. G–Q_R curves in two-terminal structures at $\Delta = 0$ (a) and 0.4 (b) with $t = t_T^{(L)} = t_T^{(R)} = 1$, $\epsilon = 0.05$ and $\Phi = 0$. In (b), the solid line corresponds to G, dashed to G_e and dotted to G_h.

3. Results and discussion

We first consider the two-terminal situation where only the left- and right-hand leads are connected to the ring. As a comparison with the results of other authors [16, 21, 23–25], the G–Q_R curve with $\Delta = 0$ is plotted in figure 2(a). (In this situation, the subscript of G_{xx} is omitted.) Here, $Q_R = \frac{Nt_{SO}}{\pi t}$, which has several physical meanings. It is the spin precession angle over the circumference of the 1D ring and is also the ratio between the perimeter and the spin precession length. According to an exact analytic expression derived from the effective mass Hamiltonian [21, 23–25], G can be written as $G = g_0(k, \Delta_{AC})(1 - \cos \Delta_{AC})$ with k the wavevector and $\Delta_{AC} = \pi\sqrt{Q_R^2 + 1}$ half the difference between the phases accumulated by spin-↑ and spin-↓ electrons. Consequently, at $Q_R = \sqrt{Z^2 - 1}$ with Z an even integer, $G = 0$, which is caused entirely by the spin-dependent quantum interference and irrelevant with the specific value of ϵ. This characteristic is verified by our numeric results. The small deviation of the zero positions from $\sqrt{Z^2 - 1}$ is due to the difference between the discrete and continuum models. As the superconducting gap is opened in the right-hand lead with $\Delta \neq 0$, the resonant peaks in the G–Q_R curve can reach 4 due to the Andreev reflection. However, the basic shape of these curves is reserved, and especially, at $Q_R = \sqrt{Z^2 - 1}$, G is still zero. At these points, an incident electron is entirely reflected, and no Cooper pair can be injected into the S lead. Consequently, no Andreev-reflected hole appears. Here and below, the energy of incident electron is set as $|\epsilon| < \Delta$ since our interest is focused on the Andreev reflection. As a result, $G_e = G_h$ at any Q_R. These results are presented in figure 2(b), where G, G_e and G_h are all plotted. To obtain them, we set $t = 1$ and $M = 100$, and these parameters are also adopted in the calculation below.

Now, we turn our attention to the four-terminal situation. At $\Delta = 0$, with the two transverse leads connected to the ring, the minimal value of G_{xx} at $Q_R = \sqrt{Z^2 - 1}$ is lifted from zero and the maximal value is lowered from 2. These changes come from two origins. First, the appearance of the two transverse leads varies $|t^{(LL)}|^2$. Second and more importantly, G_{xx} is an additional result of $-\hat{1} + |t^{(LL)}|^2$, $|t^{(LU)}|^2$ and $|t^{(LD)}|^2$. This incoherent addition will remove any resonant peak and zero conductance resulting from the spin-dependent quantum interference unless these three transmission coefficients can reach resonance or zero synchronously. Obviously, this does not happen. In the meantime, G_{sH}^z decreases in oscillation with Q_R and takes zero at $Q_R = \sqrt{Z^2 - 1}$. With the superconducting gap open in the right-hand lead, V_2 and V_3 still equal each other but they oscillate with Q_R since one of the two mirror symmetries is broken. The Andreev reflection introduces a nonzero contribution of holes, but

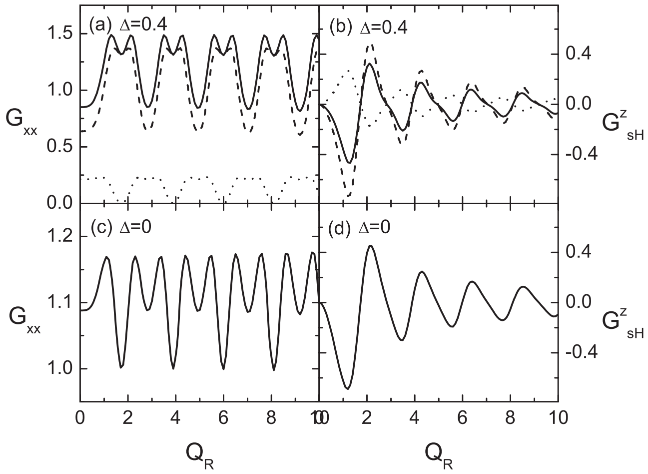

Figure 3. Variations of G_{xx} and $G^{z}_{\rm sH}$ with Q_R in four-terminal structures at $\Delta = 0.4$ ((a) and (b)) and 0 ((c) and (d)) with $t^{(L)}_{\rm T} = t^{(R)}_{\rm T} = t^{(U)}_{\rm T} = t^{(D)}_{\rm T} = 1$ and the same other parameters as in figure 2. The line texture has the same meaning as in figure 2.

$G_{{\rm e};xx} \neq G_{{\rm h};xx}$ in this four-terminal situation. Consider the e contribution of $|t^{(LU)}|^2$ as an example. It contains two parts—one comes from the direct tunnelling from U to L and the other is indirect, via the scattering at the interface between the right-hand lead and the ring—only the latter is related to the h contribution, which explains the inequality. Another noticeable fact of the longitudinal conductance is that $G_{{\rm h};xx}$ is always zero at $Q_R = \sqrt{Z^2 - 1}$ although $G_{{\rm h};xx}$ is also an additional result of $|t^{(LL)}_{\rm h}|^2$, $|t^{(LU)}_{\rm h}|^2$ and $|t^{(LD)}_{\rm h}|^2$. In fact, the Andreev reflection can happen only at the N–S interface, and if the probability of the reflected hole tunnelling from the interface to the left-hand lead is zero, these three transmission coefficients can reach zero simultaneously.

Despite the inequality $G_{{\rm e};xx} \neq G_{{\rm h};xx}$ in the four-terminal situation, the Andreev reflection still increases the total longitudinal conductance as in the two-terminal situation. However, for the spin Hall conductance, the Andreev effect reduces the total $G^{z}_{\rm sH}$, and the contribution of holes partly offsets that of electrons. This point can be seen clearly from figure 3(b), where the sign of $G^{z}_{\rm h;sH}$ is opposite to that of $G^{z}_{\rm e;sH}$ at any Q_R, analogous to the minus sign of Hall coefficient of holes in the classic Hall effect [26], but here the hole 'velocity' has the same direction as that of the electronic one, whereas in the Andreev reflection the momentum is conserved, and the 'velocity' direction of the Andreev-reflected hole is opposite to that of the incident electron. Because of the TRS of the system, a spin-down electron with the opposite 'velocity' has the same 'trajectory' as that of the spin-up electron, but moves in an opposite direction. On the other hand, with a spin-down electron transformed into a spin-up hole, it still has the same 'acceleration' if its 'velocity' remains unchanged. As a result, the Andreev-reflected spin-up hole has the same 'trajectory' as the incident spin-up electron, but moves in an opposite direction. If the incident spin-up electron induces a transverse spin current $J_{\rm z}$, the Andreev-reflected spin-up hole must induce $-J_{\rm z}$. This explains the negative contribution of Andreev-reflected holes to the spin Hall conductance. However, the h contribution cannot totally offset the e contribution, and $G^{z}_{\rm sH}$ is nonzero. In fact, if a spin-up electron is incident, say, from the left-hand lead, it can tunnel out of one of the two transverse leads via scattering at the interface between the right-hand lead and the ring but it can also tunnel out directly. Even in

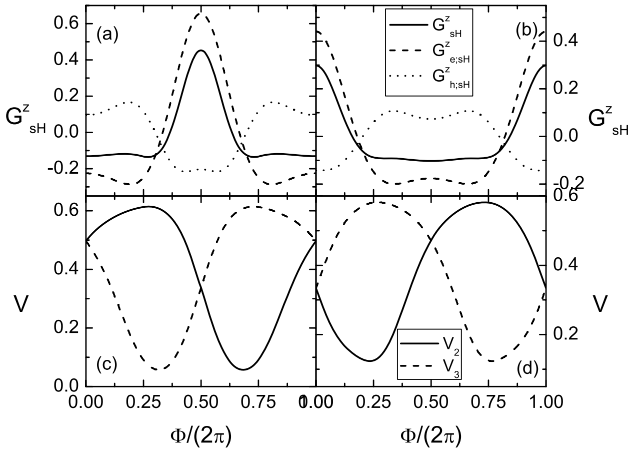

Figure 4. G^z_{sH}-Φ curves and variations of V_2 (solid) and V_3 (dashed) with Φ at $\Delta = 0.4$ with $Q_R = 0.6$ ((a) and (c)) and 2 ((b) and (d)) and the same other parameters as in figure 3.

the scattering at the N–S interface, not only is a spin-up hole reflected, but the normal reflection also occurs. These two factors result in the nonzero G^z_{sH}.

All of the above results are obtained without external magnetic flux Φ. With the TRS broken, the Andreev reflection still enhances the longitudinal conductance. These results are not presented here. In figures 4(a) and (b), the variations of G^z_{sH} with Φ are illustrated for two different values of Q_R. With a magnetic flux applied, although the Landau level and Andreev edge state [27] cannot be formed in this quasi-1D system, the classic Hall effect still plays an important role in the transport. To offset the transverse electronic current, $V_2 \neq V_3$. Due to the spin-dependent quantum interference effect in this ring structure, they oscillate with Φ as illustrated in figures 4(c) and (d). If the right lead is normal $V_2 + V_3 \equiv 1$ in the oscillation, whereas with $\Delta \neq 0$ this equality is broken. The spin-dependent quantum interference effect in this Rashba ring structure also leads to the oscillations of $G^z_{\mathrm{e;sH}}$ and $G^z_{\mathrm{h;sH}}$ with Φ. But since the Lorentz force acts only on charge, the transverse spin 'acceleration' is irrelevant with Φ. The contribution of the Andreev-reflected spin-up (down) hole to the spin Hall conductance is still opposite to that of the incident spin-up (down) electron and partly offsets the latter. Consequently, the absolute value of G^z_{sH} is reduced. Even if V_2 and V_3 are both fixed as 1/2, this basic characteristic of G^z_{sH}-Φ curves is reserved.

4. Summary

In summary, we investigate the ballistic transport properties of a mesoscopic hybrid Rashba ring and focus our attention on the influence of the Andreev reflection. This spin-nondegenerate hybrid system with four terminals is treated along the lines of BTK [19]. Although the resonant peak and zero conductance resulting from spin-dependent quantum interference are destroyed by incoherent addition in this four-terminal structure, the total longitudinal electron conductance is still enhanced by the Andreev contribution. But here $G_{\mathrm{e};xx} \neq G_{\mathrm{h};xx}$, unlike in the corresponding two-terminal system. On the other hand, the contribution of the Andreev reflection to the spin Hall conductance in this mesoscopic Rashba ring is negative to the electronic contribution and the total spin Hall conductance is reduced. These results are robust to an external magnetic flux.

Acknowledgments

The author acknowledges support by the National Foundation of Natural Science in China grant No 10204012, and by the special funds for Major State Basic Research Project No G001CB3095 of China.

References

[1] Prinz G A 1998 *Science* **282** 1660
[2] Awschalom D, Loss D and Samarth N (ed) 2002 *Semiconductor Spintronics and Quantum Computation* (Berlin: Springer)
[3] Zutic I, Fabian J and Das Sarma S 2004 *Rev. Mod. Phys.* **76** 323
[4] D'yakonov M I and Perel V I 1971 *JETP Lett.* **13** 467
[5] Hirsch J E 1999 *Phys. Rev. Lett.* **83** 1834
[6] Murakami S, Nagaosa N and Zhang S-C 2003 *Science* **301** 1348
Murakami S, Nagaosa N and Zhang S-C 2004 *Phys. Rev.* B **69** 235206
[7] Sinova J, Culcer D, Niu Q, Sinitsyn N A, Jungwirth T and MacDonald A H 2004 *Phys. Rev. Lett.* **92** 126603
[8] Rashba E I 1960 *Fiz. Tverd. Tela* **2** 1224
Rashba E I 1960 *Sov. Phys. Solid State* **2** 1109 (Engl. Transl.)
[9] Shen S-Q 2005 *Phys. Rev. Lett.* **95** 187203
[10] Nikolic B K, Zarbo L P and Welack S 2005 *Phys. Rev.* B **72** 075335
[11] Kato Y K, Myers R C, Gossard A C and Awschalom D D 2004 *Science* **306** 1910
[12] Wunderlich J, Kaestner B, Sinova J and Jungwirth T 2005 *Phys. Rev. Lett.* **94** 047204
[13] Nikolic B K, Souma S, Zarbo L P and Sinova J 2005 *Phys. Rev. Lett.* **95** 046601
[14] Sheng L, Sheng D N and Ting C S 2005 *Phys. Rev. Lett.* **94** 016602
[15] Nikolic B K, Zarbo L P and Souma S 2005 *Phys. Rev.* B **72** 075361
[16] Souma S and Nikolic B K 2005 *Phys. Rev. Lett.* **94** 106602
[17] Zhang Z-Y 2006 *J. Phys.: Condens. Matter* **18** 4101
[18] Andreev A F 1964 *Zh. Eksp. Teor. Fiz.* **46** 1823
Andreev A F 1964 *Sov. Phys.—JETP* **19** 1228 (Engl. Transl.)
[19] Blonder G E, Tinkham M and Klapwijk T M 1982 *Phys. Rev.* B **25** 4515
[20] Meijer F E, Morpurgo A F and Klapwijk T M 2002 *Phys. Rev.* B **66** 033107
[21] Souma S and Nikolic B K 2004 *Phys. Rev.* B **70** 195346
[22] Byers N and Yang C N 1961 *Phys. Rev. Lett.* **7** 46
[23] Datta S and Das B 1990 *Appl. Phys. Lett.* **56** 665
[24] Molnar B, Peeters F M and Vasilopoulos P 2004 *Phys. Rev.* B **69** 155335
[25] Frustaglia D and Richter K 2004 *Phys. Rev.* B **69** 235310
[26] Ashcroft N W and Mermin N D 1976 *Solid State Physics* (New York: Holt, Rinehart and Winston)
[27] Hoppe H, Zülicke U and Schön G 2000 *Phys. Rev. Lett.* **84** 1804